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# **Quark and Lepton Mass Matrices with a Cyclic Permutation Invariant Form**

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## **Abstract**

As an attempt to give an unified description of quark and lepton mass matrices  $M_f$ , the following mass matrix form is proposed: the form of the mass matrices are invariant under a cyclic permutation ( $f_1 \rightarrow f_2, f_2 \rightarrow f_3, f_3 \rightarrow f_1$ ) among the fermions  $f_i$ . The model naturally leads to the maximal mixing between  $\nu_\mu$  and  $\nu_\tau$ , and with an additional ansatz, it leads to the well-satisfied relations  $|V_{us}| \simeq \sqrt{m_d/m_s}$  and  $|V_{cb}| \simeq \sqrt{m_d/m_b}$ .

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# 1 Introduction

The times of the phenomenological study of the quark mass matrices have been over. However, as far as the study of the neutrino mass matrix is concerned, the phenomenological study still have the meaning. It is still important to investigate the unified description of the quark and lepton mass matrices from the phenomenological point of view.

In such a phenomenological study, the key to the unified description is to find a fermion basis on which the quark and lepton mass matrices at the unification energy scale take a simpler and beautiful form. Concerning this point, recently, Takasugi and his collaborators [1] have discussed a neutrino mass matrix on a very interesting basis, which is described by a discrete symmetry  $Z_3$ . However, in their neutrino mass matrix model, the charged lepton mass matrix is given by a diagonal form, so that the model cannot give any predictions for charged lepton mass spectrum. And, at present, they have not applied their idea to the quark mass matrices. However, their basic idea seems to be highly promising for a unified description of quark and lepton mass matrices.

Stimulated by their idea, in the present paper, we investigate fermion mass matrices with the form

$$M = aE + bS(\theta), \quad (1.1)$$

$$E = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad S(\theta) = \frac{1}{\sqrt{6}} \begin{pmatrix} 0 & e^{i\theta} & e^{-i\theta} \\ e^{-i\theta} & 0 & e^{i\theta} \\ e^{i\theta} & e^{-i\theta} & 0 \end{pmatrix}, \quad (1.2)$$

where the matrices  $E$  and  $S(\theta)$  have been normalized as  $\text{Tr}E^2=\text{Tr}S^2(\theta)=1$ . Although Takasugi and his collaborators [1] have related the form (1.1) to a  $Z_3$  symmetry, in this paper, we require that the mass matrix is invariant under a cyclic permutation ( $f_1 \rightarrow f_2, f_2 \rightarrow f_3, f_3 \rightarrow f_1$ ) where  $f_i$  are quarks and lepton fields ( $f = u, d, \nu, e$ ). The form (1.1) is the most general form which is invariant under the cyclic permutation and which is Hermitian. The matrix  $M$  given in Eq. (1.1) is diagonalized by the tri-maximal mixing matrix [2]

$$V_T \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{pmatrix}, \quad (1.3)$$

where  $\omega = e^{2\pi i/3}$ , as follows:

$$V_T M V_T^\dagger = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \quad (1.4)$$

$$\begin{aligned}
m_1 &= \frac{1}{\sqrt{3}}a + \frac{2}{\sqrt{6}}b \cos \theta, \\
m_2 &= \frac{1}{\sqrt{3}}a - \frac{1}{\sqrt{6}}b \cos \theta + \frac{1}{\sqrt{2}}b \sin \theta, \\
m_3 &= \frac{1}{\sqrt{3}}a - \frac{1}{\sqrt{6}}b \cos \theta - \frac{1}{\sqrt{2}}b \sin \theta,
\end{aligned} \tag{1.5}$$

Note that any mass spectrum with three families can be described by the form given in Eq. (1.1), because the three terms ( $a$ -,  $b \cos \theta$ -, and  $b \sin \theta$ -terms) in Eq. (1.1) are transformed into the three diagonal matrices,  $\text{diag}(1, 1, 1)$ ,  $\text{diag}(2, -1, -1)$ , and  $\text{diag}(0, 1, -1)$ , respectively, which are independent of each other. Therefore, we have the same number of parameters as the number of the observable quantities (mass values). Besides, if the mass matrices of all fermion sectors are given by the form (1.1), then the Cabibbo-Kobayashi-Maskawa (CKM) matrix  $V$  will be given by  $V = V_T V_T^\dagger = \mathbf{1}$  (or  $V = V_T^2$ ). We must assume an additional term which will break the cyclic permutation symmetry. Nevertheless, in this paper, we would like to emphasize that the mass matrix form (1.1) which is given on the cyclic permutation symmetric basis will shed a new light on the unified description of quarks and leptons. The purpose of the present paper is not to give a theoretical model for the unified description, but to show how we can see many suggestive empirical relations for the fermion masses and mixings if we take a basis on which the mass matrices are cyclic permutation invariant.

## 2 Basic assumptions

Our basic assumptions for the fermion mass matrices  $M_f$  are as follows: (i) The mass matrices  $M_f$  are given by a bi-linear form

$$M_f = m_0^f K_f K_f^\dagger, \tag{2.1}$$

and (ii) the matrices  $K_f$  have a cyclic permutation invariant form

$$K_f = a_f E - b_f S(\theta_f), \tag{2.2}$$

where  $E$  and  $S(\theta_f)$  are defined in Eq. (1.2). Here, in contrast to the definition of  $M$  given in Eq. (1.1), we have changed the sign of the coefficient of  $S(\theta)$  such as the angle  $\theta_f$  is in the range  $0 \leq \theta_f \leq \pi/2$  for  $a_f > 0$ ,  $b_f > 0$  and  $m_1^f < m_2^f < m_3^f$ . In the expression (2.2), the substantial parameters are only  $b_f/a_f$  and  $\theta_f$ , because we discuss only the mass ratios in the present paper.

The form of  $M_f$ , (2.1), may be interpreted by a generalized seesaw scenario [3]

$$(\bar{f}_L \ \bar{F}_L) \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} \begin{pmatrix} f_R \\ F_R \end{pmatrix}, \quad (2.3)$$

with  $m_L \propto m_R \propto K_f$  and  $M_F \propto \mathbf{1}$ , where  $F$  are hypothetical heavy fermions in addition to the conventional quarks and leptons  $f$ . However, in the present paper, we do not discuss the origin of the form of  $M_f$  given in Eq. (2.1) and confine ourselves to discuss phenomenological aspects of the model.

Note that if we assume  $b_f/a_f = 1$ , we obtain the relation [5]

$$m_1 + m_2 + m_3 = \frac{2}{3}(\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3})^2, \quad (2.4)$$

which is excellently satisfied by the observed charged lepton masses.

In Table 1, we give values of  $b_f/a_f$  and  $\theta_f$  which are evaluated from the observed values of  $m_i^f$  and the relations

$$R \equiv 3 \frac{m_1 + m_2 + m_3}{(\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3})^2} = 1 + (b_f/a_f)^2, \quad (2.5)$$

and

$$\tan \theta_f = \sqrt{3} \frac{\sqrt{m_3} - \sqrt{m_2}}{\sqrt{m_3} + \sqrt{m_2} - 2\sqrt{m_1}}. \quad (2.6)$$

For comparison, we have given the values  $b_f/a_f$  and  $\theta_f$  for the following three cases: (a) the observed charged lepton masses and the running quark masses  $m_i^q(\mu)$  at  $\mu = m_Z$ , (b) the fermion masses  $m_i^f(\mu)$  at  $\mu = \Lambda_X = 2 \times 10^{16}$  GeV in a non-SUSY scenario, and (c) the fermion masses  $m_i^f(\mu)$  at  $\mu = \Lambda_X = 2 \times 10^{16}$  GeV in a SUSY scenario (the quark mass values have been quoted from [4]). As seen in Table 1, the values  $b_f/a_f$  and  $\theta_f$  are not so sensitive to the renormalization group effects (evolution of the Yukawa coupling constants), because those have been determined only from mass ratios.

We may read the values  $\theta_f$  given in Table 1 as

$$\cos \theta_e = \sqrt{\frac{11}{24}}, \quad \cos \theta_d = \sqrt{\frac{9}{24}}, \quad \cos \theta_u = \sqrt{\frac{7}{24}}, \quad (2.7)$$

which give the angle values  $\theta_e = 47.39^\circ$ ,  $\theta_d = 52.24^\circ$  and  $\theta_u = 57.31^\circ$ , respectively, and give the relation  $\cos^2 \theta_e - \cos^2 \theta_d = \cos^2 \theta_d - \cos^2 \theta_u = 1/12$ . Of course, the relations (2.7) may be accidental coincidence, and they do not need to be taken seriously.

Note that not only the down-lepton masses, but also the down-quark masses give  $b_f/a_f \simeq 1$ , so that the down-quark masses also satisfy the relation given in Eq. (2.4). [However, the value of  $m_1^f/m_2^f$  is sensitive to the deviation of  $b_f/a_f$  from  $b_f/a_f = 1$ . If we take  $b_d/a_d = 1$ , we must accept the prediction of  $m_1^d$  with the deviation of  $2\sigma$ .] On the other hand, in contrast to the values of  $b_e/a_e$  and  $b_d/a_d$ , the value of  $b_u/a_u$  is considerably deviated from  $b_u/a_u = 1$ . For a reference, in Table 1, we list values of  $\tan^{-1}(b_f/a_f)$ , where we put  $a_f$  and  $b_f$  as  $a_f = \cos \phi_f$  and  $b_f = \sin \phi_f$  since only the ratio  $b_f/a_f$  is meaningful. It is interesting that  $\phi_f$  show  $\phi_u \simeq \theta_d$  and  $\phi_d \simeq \theta_e$  and  $\phi_e = 45^\circ$ .

### 3 Neutrino mass matrix

We assume that neutrino mass matrix  $M_\nu$  is generated by the seesaw mechanism[6], i.e.,

$$M_\nu \simeq -M_D M_R^{-1} M_D^T, \quad (3.1)$$

where  $M_D$  and  $M_R$  are Dirac and Majorana mass matrices,  $\bar{\nu}_L M_D \nu_R$  and  $\bar{\nu}_R^c M_R \nu_R$ , respectively, where  $\nu_R^c = (\nu_R)^c = C \nu_R^T$ . Since we consider that in a similar way to the charged lepton sector, the Dirac mass matrix  $M_D$  is diagonalized by the tri-maximal mixing matrix  $V_T$  as

$$V_T M_D V_T^\dagger = D_D = \text{diag}(m_1^D, m_2^D, m_3^D). \quad (3.2)$$

The neutrino mass matrix  $M_\nu$  given in Eq. (3.1) is written as

$$M'_\nu = V_T M_\nu V_T^T \simeq -D_D (V_T^* M_R V_T^\dagger)^{-1} D_D. \quad (3.3)$$

If the Majorana mass matrix  $M_R$  is simply given by  $M_R = m_R \mathbf{1}$ , then, by using the relation

$$V_T V_T^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (3.4)$$

we obtain the form

$$M'_\nu = -\frac{1}{m_R} D_D \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}^{-1} D_D = -\frac{1}{m_R} \begin{pmatrix} (m_1^D)^2 & 0 & 0 \\ 0 & 0 & m_2^D m_3^D \\ 0 & m_2^D m_3^D & 0 \end{pmatrix}, \quad (3.5)$$

When we take  $(m_1^D)^2 \simeq m_2^D m_3^D$ , we obtain  $m_1^\nu \simeq |m_2^\nu| = |m_3^\nu| = m_2^D m_3^D / m_R$  and  $\sin^2 2\theta_{23} = 1$ . Thus, we can obtain a natural explanation of the maximal mixing between  $\nu_\mu$  and  $\nu_\tau$

which is suggested by the atmospheric neutrino data [7]. However, the present oversimplified scenario ( $M_R = m_R \mathbf{1}$ ) cannot give  $|\Delta m_{12}^2| \ll |\Delta m_{23}^2|$  ( $\Delta m_{ij}^2 = (m_i^\nu)^2 - (m_j^\nu)^2$ ) and  $\sin^2 2\theta_{12} \neq 0$  which are required for the explanation of the solar neutrino data [8]. In order to give a realistic numerical result, we must take other terms in  $M_R$  and  $M_D$  into consideration.

For the structure of  $M_R$ , an alternative scenario is also attractive : the Majorana mass term  $M'_R = V_T M_R V_T^T$  on the basis  $\nu'_R = V_T \nu_R$  is again given by the cyclic permutation invariant form

$$M'_R = m_R K_R. \quad (3.6)$$

[However,  $M'_R$  is invariant under the cyclic permutation not of  $\nu_{Ri}$ , but of  $\nu'_R$ .] Since  $M'_R$  must be symmetric, i.e.,  $(M'_R)^T = M'_R$ , the angle parameter  $\theta$  must be zero :

$$K_R = a_R E - b_R S(0). \quad (3.7)$$

If we take a special case  $b_R/a_R = \sqrt{2}$ , i.e.,

$$K_R = \frac{a_R}{\sqrt{3}} \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}, \quad (3.8)$$

we obtain

$$(M'_R)^{-1} = -\frac{1}{m_R} \frac{\sqrt{3}}{2a_R} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad (3.9)$$

and

$$M'_\nu \simeq -D_D (M'_R)^{-1} D_D = \frac{1}{m_R} \frac{\sqrt{3}}{2a_R} \begin{pmatrix} 0 & m_1^D m_2^D & m_1^D m_3^D \\ m_1^D m_2^D & 0 & m_2^D m_3^D \\ m_1^D m_3^D & m_2^D m_3^D & 0 \end{pmatrix}. \quad (3.10)$$

The form of  $M'_\nu$  in Eq. (3.10) is already discussed in Ref. [9] for the case  $b_\nu = -1/2$  in the “democratic seesaw” model [10]

$$M_f \simeq -m_L M_F^{-1} m_R, \quad (3.11)$$

$$m_L = \frac{1}{\kappa} m_R = \frac{m_0}{\sqrt{m_\tau + m_\mu + m_e}} \text{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau}), \quad (3.12)$$

$$M_F = m_F \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + b_f \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \right\}, \quad (3.13)$$

and it is known [9] that the case  $b_\nu = -1/2$  gives the maximal mixing between  $\nu_\mu$  and  $\nu_\tau$ .

Furthermore, if we consider a special case with  $m_2^D = m_3^D$ , which arises from  $\theta_D = 0$  in  $M_D = K_D K_D^\dagger$ , we obtain a simple neutrino mass matrix of the form

$$M'_\nu = \begin{pmatrix} 0 & x & x \\ x & 0 & y \\ x & y & 0 \end{pmatrix}, \quad (3.14)$$

where  $x = (\sqrt{3}/2a_R m_R) m_1^D m_3^D$  and  $x/y = m_1^D/m_3^D$ . This matrix form given in Eq. (3.14) has recently been proposed by Ghosal [11] on the basis of discrete  $Z_3 \times Z_4$  symmetries. The mass matrix  $M'_\nu$  given in Eq. (3.14) gives the eigenvalues  $(m_1^\nu, -m_2^\nu, -m_3^\nu)$ ,

$$m_1^\nu = \frac{1}{2}(\sqrt{y^2 + 8x^2} + y), \quad -m_2^\nu = -\frac{1}{2}(\sqrt{y^2 + 8x^2} - y), \quad -m_3^\nu = -y, \quad (3.15)$$

(we have defined  $m_i^\nu$  as  $m_1^\nu > 0$  and  $m_1^\nu > m_2^\nu > m_3^\nu$ ), and the mixing matrix

$$U = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{1}{\sqrt{2}}s_{12} & \frac{1}{\sqrt{2}}c_{12} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}s_{12} & \frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (3.16)$$

where  $c_{12} = \cos \theta_{12} = \sqrt{m_1^\nu/(m_1^\nu + m_2^\nu)}$  and  $s_{12} = \sin \theta_{12} = \sqrt{m_2^\nu/(m_1^\nu + m_2^\nu)}$ . Therefore, for  $x \gg y$  ( $|m_1^D| \gg |m_3^D|$ ), we obtain the relations  $\Delta m_{12}^2 \simeq 2\sqrt{2}xy$  and  $\Delta m_{23}^2 \simeq 2x^2$  together with  $c_{12} \simeq s_{12} \simeq 1/\sqrt{2}$ , which leads to a bi-maximal mixing. Thus, we can give a reasonable explanation both for the atmospheric and solar neutrino data regarding  $\Delta m_{12}^2 \simeq \Delta m_{solar}^2 \sim 10^{-10} \text{ eV}^2$  and  $\Delta m_{23}^2 \simeq \Delta m_{atm}^2 \sim 10^{-3} \text{ eV}^2$ .

## 4 CKM mixing matrix

If we apply the matrix form given in Eq. (2.2) to the up- and down-quark sectors, we obtain the wrong result  $V = \mathbf{1}$ . Therefore, let us modify the matrix  $K_f$  by adding a small term  $c_f P_\omega$ :

$$K_f = a_f E - b_f S(\theta_f) + c_f P_\omega. \quad (4.1)$$

$$P_\omega = \frac{1}{\sqrt{3}} \text{diag}(\omega, \omega^2, 1) . \quad (4.2)$$

Then, the form (4.1) is not invariant under the cyclic permutation ( $f_1 \rightarrow f_2, f_2 \rightarrow f_3, f_3 \rightarrow f_1$ ). The term  $(\omega \bar{f}_{1L} f_{1R} + \omega^2 \bar{f}_{2L} f_{2R} + \bar{f}_{3L} f_{3R})$  is transformed into  $\omega^2 (\omega \bar{f}_{1L} f_{1R} + \omega^2 \bar{f}_{2L} f_{2R} + \bar{f}_{3L} f_{3R})$  under the cyclic permutation. However, if the  $E$  and  $S$  terms are absent in the matrix  $K_f$ , the form of  $P_\omega$  is invariant under the cyclic permutation, because the common phase factor is unphysical. We regard the small  $P_\omega$  term as a “form” invariant term under the cyclic permutation in the extended meaning.

The modified matrix  $K_f$  given in Eq. (4.1), is transformed by the tri-maximal mixing matrix  $V_T$  as

$$K'_f = V_T K_f V_T^\dagger = \frac{1}{\sqrt{3}} \begin{pmatrix} \lambda_1^f & 0 & 0 \\ 0 & \lambda_2^f & 0 \\ 0 & 0 & \lambda_3^f \end{pmatrix} + \frac{c_f}{\sqrt{3}} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad (4.3)$$

where  $\lambda_i^f$  are the eigenvalues of the matrix  $\sqrt{3}[a_f E - b_f S(\theta_f)]$  and they are explicitly given in Eq. (1.5) with  $m_i \rightarrow \lambda_i/\sqrt{3}$ .

The mixing matrix  $U_L$  is obtained by the diagonalization of the Hermitian matrix

$$M'_f = m_0 K'_f (K'_f)^\dagger = \frac{1}{3} m_0 \left[ \begin{pmatrix} \lambda_1^2 & \lambda_2 c_f & \lambda_1 c_f^* \\ \lambda_2 c_f^* & \lambda_2^2 & \lambda_3 c_f \\ \lambda_1 c_f & \lambda_3 c_f^* & \lambda_3^2 \end{pmatrix} + |c_f|^2 \mathbf{1} \right]. \quad (4.4)$$

For a small  $c_f$  and  $\lambda_3^2 \gg \lambda_2^2 \gg \lambda_1^2$ , the eigenvalues of  $M'_f$  are given by

$$m_1 \simeq \frac{1}{3} m_0 \lambda_1^2, \quad m_2 \simeq \frac{1}{3} m_0 (\lambda_2^2 + |c_f|^2), \quad m_3 \simeq \frac{1}{3} m_0 (\lambda_3^2 + 2|c_f|^2), \quad (4.5)$$

and the mixing angles are given by

$$\tan 2\theta_{12} \simeq \frac{2\lambda_2 |c_f|}{\lambda_2^2 - \lambda_1^2} \simeq 2 \frac{|c_f|}{\lambda_2}, \quad (4.6)$$

$$\tan 2\theta_{23} \simeq \frac{2\lambda_3 |c_f|}{\lambda_3^2 - \lambda_1^2} \simeq 2 \frac{|c_f|}{\lambda_3}, \quad (4.7)$$

and  $\theta_{13} \simeq 0$ . (If  $c_f$  is complex, i.e.,  $\arg c_f \neq 0$ , the elements  $U_{12} = \sin \theta_{12}$  and  $U_{23} = \sin \theta_{23}$  are replaced by  $U_{12} = \sin \theta_{12} \exp(i \arg c_f)$  and  $U_{23} = \sin \theta_{23} \exp(i \arg c_f)$ , respectively.)



If we consider that the value of  $|c_f|^2$  in the up-quark sector is sufficiently small so that  $\theta_{ij}^u \simeq 0$ , we obtain

$$\left| \frac{V_{cb}}{V_{us}} \right| \simeq \frac{|U_{23}^d|}{|U_{12}^d|} \simeq \frac{\lambda_2^d}{\lambda_3^d} \simeq \sqrt{\frac{m_s}{m_b}} = 0.176, \quad (4.8)$$

which is in good agreement with the experimental value [12]  $|V_{cb}/V_{us}| = (0.0373 \pm 0.0018)/(0.2205 \pm 0.0018) = 0.169 \pm 0.008$ . Note that in order to obtain the relation (4.8), the assumption (i) given in Eq. (2.1) is essential.

When we denote  $M'_f$  as

$$M'_f = M_f^0 + \frac{1}{3}|c_f|^2 \mathbf{1}, \quad (4.9)$$

as a trial, let us suppose that the lowest one of the three eigenvalues of the mass matrix  $M_f^0$  except for the common constant term  $|c_f|^2 \mathbf{1}$  is assigned to zero, i.e.,

$$\det M_f^0 = 0. \quad (4.10)$$

In this scenario, we obtain

$$|c_f|^2 \simeq \lambda_1^2, \quad (4.11)$$

together with  $m_1^f = |c_f|^2/3$ . Neglecting the mixings in the up-quark sector, we obtain

$$|V_{us}| \simeq |c_d|/\lambda_2^d \simeq \sqrt{m_d/m_s} = 0.224, \quad (4.12)$$

$$|V_{cb}| \simeq |c_d|\lambda_3^d \simeq \sqrt{m_d/m_b} = 0.0395, \quad (4.13)$$

which are in good agreement with the experiments. However, at present, there is no theoretical understanding behind the assumption given in Eq. (4.10).

The existence of the  $P_\omega$  term with  $|c_f|^2 \simeq (\lambda_1^f)^2$  slightly changes the predicted values of  $m_i^f$  from the case of  $c_f = 0$ . Especially the values of  $m_2^f$  is sizably changed as  $m_2^f = (\lambda_2^f)^2/3 \rightarrow m_2^f \simeq [(\lambda_2^f)^2 + |c_f|^2]/3$ . However, for the charged leptons, the existence dose not so badly spoil the relation given in Eq. (2.4), because  $|c_e|^2/(\lambda_2^e)^2 \simeq (\lambda_1^e/\lambda_2^e)^2 \simeq (m_1^e/m_2^e) \sim 10^{-3}$ . In order to obtain the best fit values of  $m_i^f$ , the values of  $b_f/a_f$  and  $\theta_f$  will slightly be changed from the values given in Table 1.

## 5 Concluding remarks

In conclusion, we have pointed out that we can see many interesting relations for the quark and lepton masses and their mixings if we take a basis on which the form of the mass matrices is invariant under the cyclic permutation ( $f_1 \rightarrow f_2, f_2 \rightarrow f_3, f_3 \rightarrow f_1$ ). We have assumed that the mass matrices  $M_f$  are given by the bi-linear form [given in Eq. (2.1)],

where the matrix  $K_f$  is given by Eq. (4.1). If we put  $b_e/a_e = 1$  together with  $c_e = 0$ , we obtain the relation given in Eq. (2.4) which is in excellent agreement with experiments. We think that the form of the down-lepton mass matrix with  $b_f/a_f = 1$  is the most fundamental one. Although there is at present no explicit theoretical ground for the ansatz given in Eq. (4.10), however, if we accept the ansatz, we can obtain the well-satisfied relations for  $|V_{us}| \simeq \sqrt{m_d/m_s}$  and  $|V_{cb}| \simeq \sqrt{m_d/m_b}$ . It is also worth while to note that the model naturally leads to the maximal mixing between  $\nu_\mu$  and  $\nu_\tau$ . (This result is independent of the assumption given in Eq. (2.1).) If we take the special values of the parameters, we can obtain a beautiful neutrino mass matrix (Ghosal's neutrino mass matrix) given in Eq. (3.14) which leads to bi-maximal mixing.

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Table 1: Numerical values of the parameters  $b_f/a_f$  [and also  $\phi_f = \tan^{-1}(b_f/a_f)$ ] and  $\theta_f$ , which are defined by (2.5) and (2.6) in the text. Input values  $m_i^f(\mu)$  are quoted from Ref. [4], where  $\Lambda_X = 2 \times 10^{16}$  GeV. The values of  $b_f/a_f$  and  $\theta_f$  have been evaluated only from the center values of  $m_i^f(\mu)$ .

	Inputs			Outputs		
	$m_1^f(\text{MeV})$	$m_2^f(\text{MeV})$	$m_3^f(\text{GeV})$	$b_f/a_f$	$\phi_f$	$\theta_f$
$(m_i^e)_{obs}$	0.510999	105.6584	$1.77705^{+0.00029}_{-0.00029}$	1.0000	$45.000^\circ$	$47.2680^\circ$
$m_i^e(\Lambda_X)_{non-SUSY}$	0.493486	104.152	$1.7706 \pm 0.0003$	1.0019	$45.054^\circ$	$47.336^\circ$
$m_i^e(\Lambda_X)_{SUSY}$	0.3250203	68.598	$1.1714 \pm 0.0002$	1.0026	$45.073^\circ$	$47.367^\circ$
$m_i^d(m_Z)$	$4.69^{+0.60}_{-0.66}$	$93.4^{+11.8}_{-13.0}$	$3.00 \pm 0.11$	1.047	$46.30^\circ$	$52.43^\circ$
$m_i^d(\Lambda_X)_{non-SUSY}$	$1.49^{+0.25}_{-0.28}$	$38.7^{+4.9}_{-5.4}$	$1.07 \pm 0.04$	1.024	$45.69^\circ$	$51.77^\circ$
$m_i^d(\Lambda_X)_{SUSY}$	$1.33^{+0.17}_{-0.19}$	$26.5^{+3.3}_{-3.7}$	$1.00 \pm 0.04$	1.070	$46.93^\circ$	$53.07^\circ$
$m_i^u(m_Z)$	$2.33^{+0.42}_{-0.45}$	$677^{+56}_{-61}$	$181 \pm 13$	1.287	$52.15^\circ$	$57.05^\circ$
$m_i^u(\Lambda_X)_{non-SUSY}$	$0.94^{+0.17}_{-0.18}$	$272^{+22}_{-24}$	$84^{+18}_{-13}$	1.295	$52.33^\circ$	$57.26^\circ$
$m_i^u(\Lambda_X)_{SUSY}$	$1.04^{+0.19}_{-0.20}$	$302^{+25}_{-27}$	$129^{+196}_{-40}$	1.312	$52.68^\circ$	$57.68^\circ$